Faculty
Of
Computing

SEMESTER I 2024/2025

SECI1013
Discrete Structure I

Assignment 1.2

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Question 1

(1) $A = \{3, 6, 9, 12\}$
$B = \{2, 3, 4, 5, 6\}$

(i) $\Rightarrow$ $a - b$ is an even integer, $a \in A$, $a \in B$

Test pairs:

$a = 3$, $a - b = \text{even}$

$3 - 2 = 1$ (odd)
$3 - 3 = 0$ (even)
$3 - 4 = -1$ (odd)
$3 - 5 = -2$ (even)
$3 - 6 = -3$ (odd)

pairs for $a = 3$: $(3,3)$, $(3,5)$.

$a = 6$,

$6 - 2 = 4$ (even)
$6 - 3 = 3$ (odd)
$6 - 4 = 2$ (even)
$6 - 5 = 1$ (odd)
$6 - 6 = 0$ (even)

pairs for $a = 6$: $(6,2)$, $(6,4)$, $(6,6)$

$a = 9$,

$9 - 2 = 7$ (odd)
$9 - 3 = 6$ (even)
$9 - 4 = 5$ (odd)
$9 - 5 = 4$ (even)
$9 - 6 = 3$ (odd)

pairs for $a = 9$: $(9,3)$, $(9,5)$

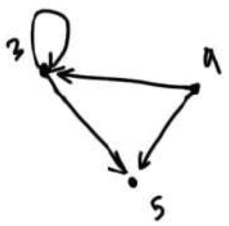
$a = 12$,

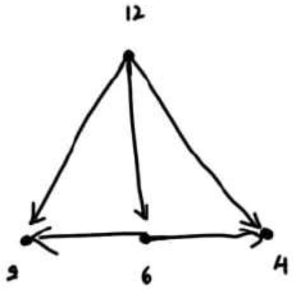
$12 - 2 = 10$ (even)
$12 - 3 = 9$ (odd)
$12 - 4 = 8$ (even)
$12 - 5 = 7$ (odd)
$12 - 6 = 6$ (even)

pairs for $a = 12$: $(12,2)$, $(12,4)$, $(12,6)$

$R = \{(3,3), (3,5), (6,2), (6,4), (6,6), (9,3), (9,5), (12,2), (12,4), (12,6)\}$.

(ii) $A = \{3, 6, 9, 12\}$ $B = \{2, 3, 4, 5, 6\}$





(iii) Domain $= \{3, 6, 9, 12\}$

Range $= \{2, 3, 4, 5, 6\}$

Question 2

Question 2

Determine whether the relation on set $D = \{1, 3, 8, 10, 15\}$ is equivalent relations when $x, y \in D$, xRy if and only if $y-x$ is a multiple of 7 (including negative)

Answers

$D = \{1, 3, 8, 10, 15\}$

$x, y \in D$, xRy if and only if $y-x$ is a multiple 7 (including negative)

possible element of set R :-

$y-x = 1-8 = -7$ (multiple 7)
(8,1)
$y-x = 15-8 = 7$ (multiple 7)
(8,15)
$y-x = 3-10 = -7$ (multiple 7)
(10,3)
$y-x = 10-3 = 7$ (multiple 7)
(3,10)

$y-x = 8-1 = 7$ (multiple 7)
(1,8)
$y-x = 15-1 = 14$ (multiple 7)
(1,15)
$y-x = 1-15 = -14$ (multiple 7)
(15,1)
$y-x = 8-15 = 7$ (multiple 7)
(15,8)
$y-x = 1-1 = 0$ (multiple 7)
(1,1)

$y-x = 3-3 = 0$ (multiple 7)
(3,3)
$y-x = 8-8 = 0$ (multiple 7)
(8,8)
$y-x = 10-10 = 0$ (multiple 7)
(10,10)
$y-x = 15-15 = 0$ (multiple 7)
(15,15)

Therefore elements of R

is $R = \{(8,1)\ (8,15)\ (10,3), (3,10), (1,8), (1,15), (15,8), (1,1), (3,3)$
$(8,8), (10,10), (15,15), (15,1)\}$

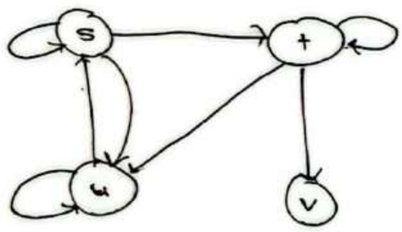
- Based on elements R, R is symmetric because every $(x,y) \in R$, $(y,x) \in R$
- Based on elements R, R is reflexive because some of element R has $(x,y) \in R$, $x = y$ which is (1,1), (3,3), (8,8), (15,15).

$$
\begin{array}{c} \\ 1 \\ 3 \\ 8 \\ 10 \\ 15 \end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 3 & 8 & 10 & 15 \end{array} \\
\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}
\end{array}
\otimes
\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}
=
\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}
$$

- Relation R is a transitive
- In conclusion, R is equivalent relations

Question 3

3. Given the diagraph of relation R as in Figure 1.



i) What is matric of the relation, M_R that represent diagraph in Figure 1

$$M_R = \begin{array}{c} \\ s \\ u \\ t \\ v \end{array} \begin{array}{c} \begin{array}{cccc} s & u & t & v \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(ii) List in degrees and out-degrees of all vertices

s (in-degrees) = 2 t (in-degrees) = 2

s (out-degrees) = 3 t (out-degrees) = 3

u (in-degrees) = 3 v (in-degrees) = 1

u (out-degrees) = 2 v (out-degrees) = 0

(iii) Is it the relation of R is an partial order? Check all variance. Justify for answers.

- Characteristic of partial order relation is reflexsive, antisymmetric and transitive
- from the matrix of relation R, we know that R is not reflexsive because there is no (v,v) in R
- We also know R is not antisymmetric because there is (s,u), (u,s) ∈ R
- lastly we determine is not transitive as below

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

∴ Therefore, relation R is not partial order

Question 4

Question 4

4. Let $X = (-2, 0, 2)$ and $Y = (-4, 0, 4)$. For each $x \in X$, define function $v: X \to Y$ and $w: X \to Y$ by:

$v(x) = 4 - x^2$

$w(x) = 2x$

Determine if v and w are one-to-one, onto Y, and/or bijection

~~For fun~~

Answer

For function v

$v(x) = 4 - x^2$

when $x = -2$: $v(-2) = 4 - (-2)^2 = 0$

when $x = 0$: $v(0) = 4 - (0)^2 = 4$

when $x = 2$: $v(2) = 4 - (2)^2 = 0$

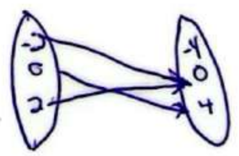
· For function w

$w(x) = 2x$

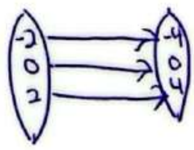
$w(2) = 2(2) = 4$

$w(0) = 2(0) = 0$

$w(-2) = 2(-2) = -4$



X V Y



X W Y

Conclusion of v function :-

- v function is not one-to-one function because $v(2)$ and $v(-2)$ is equal to 0
- v function is not onto Y because v function do not cover all elements Y

= Therefore v function is not bijection function

conclusion of

- w function is one-to-one function because they all have unique value from elements X to elements Y
- w function is onto Y because its cover all element of Y.

- Therefore, w function is bijection function

Queetion 5

(5) $f(x) = 7x-2$

$g(x) = \frac{2}{3}x$

(i) inverse of $g^{-1}(x)$

$g(x) = \frac{2}{3}x$

let $y = g(x)$

$y = \frac{2}{3}x$

$3y = 2x$

$\frac{3}{2} = x$

$g^{-1}(x) = \frac{3}{2}x$

(ii) $(g \circ g \circ f)(x)$

$f(x) = 7x-2$

$g(x) = \frac{2}{3}x$

$g(g(f(x)))$

$g(f(x)) = g(7x-2)$

$= \frac{2}{3}(7x-2)$

$= \frac{14}{3}x - \frac{4}{3}$

$g(g(f(x))) = g\left(\frac{14}{3}x - \frac{4}{3}\right)$

$= \frac{2}{3}\left(\frac{14}{3}x - \frac{4}{3}\right)$

$= \frac{28}{9}x - \frac{8}{9}$

$\therefore g \circ g \circ f(x) = \frac{28}{9}x - \frac{8}{9}$

Question 6

6 (i) initial T for chemical A is F_0 which is 5.0

initial T for chemical B is F_1 which is 4.5

F_t is T for chemical C

$F_t = F_{t-1} + 0.2(F_{t-2})$, $t \geq 2$, $F_0 = 5.0$, $F_1 = 4.5$ #

(ii) $F_0 = 5$, $F_1 = 4.5$

from the recurrence of chemical C , we can substitute the value

$F_2 = 4.5 + 0.2(5.0)$

$F_2 = 5.5$

$F_3 = 5.5 + 0.2(4.5)$
$= 6.4$

$F_4 = 6.4 + 0.2(5.5)$
$= 7.5$

$F_5 = 7.5 + 6.4(0.2)$
$= 8.78$

Question 7

Question 7

Write a recursive algorithm to find the n term of the sequence defined $W_0 = 5$, $W_1 = 7$ and $W_n = 2W_{n-1} + W_{n-2}$ for $n \geq 2$. Trace the algorithm for n=4.

Answer

recursive algorithm :-

```
W(n)
  { if (n=0)
     return 5
    else if (n=1)
     return 7
    return 2W(n-1) + W(n-2)
  }
```

Trace the output for n=4

W(4)
n=4
because n≠1 and n≠0
return 2W(3) + W(2) — return 2(45)+19
w(4) = 109

W(3)
n=3
because n≠1 and n≠0
return 2W(2) + W(1) — return 2(19)+7
W(3) = 45

W(2)
n=2
because n≠1 and n≠0
return 2W(1) + W(0) — return 2(7) +5
W(2) = 19

W(1)
n=1
because n=1
return 7 — return 7
W(1) = 7

W(0)
n=0
because n=0
return 5 — return 5
W(0) = 5

Based on tracing algorithm ,
w(2)= 19,w(3) =45 , w(4) =109